Review of M. Fournié, J.-Ph. Furter, D. Pinchon, "Computation of the maximal degree of the inverse of a cubic automorphism of the affine plane with Jacobian 1 via Gröbner bases," J. Symbolic Computation **26** (1998), 381–386.

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Let p, q, r, s, t, u be elements of a $\mathbf{Q}$-algebra K. Consider the polynomials $f = x + px^2 + qxy + ry^2$ and $g = y + sx^2 + txy + uy^2$ in $K[x, y]$. One may write x as a series $f - pf^2 - qfg - rg^2 + (2p^2 + qs)f^3 + \cdots$ in f and g. If the Jacobian determinant $J(f, g) = (\partial f/\partial x)(\partial g/\partial y) - (\partial f/\partial y)(\partial g/\partial x)$ is equal to 1 then this series has only finitely many nonzero terms: $x = f - pf^2 - qfg - rg^2$. One can mechanically verify this formula for x by substitution.

This paper displays the results of the same computations for cubics. It considers polynomials f and g of degree at most 3 such that $J(f, g) = 1$, and expresses x as a polynomial $\sum_{i+j\le 9} c_{i,j} f^i g^j$, the coefficients $c_{i,j}$ being specific elements of $\mathbf{Q}[p, q, \ldots]$ reduced modulo some Gröbner basis for the ideal generated by the coefficients of $J(f, g) - 1$. Here 9 is smallest possible. (The reduced polynomials occupy more than two pages and have not been checked by this reviewer.)

Given similar numerical data for quartics, quintics, etc., one might try to formulate a quantitative generic version of the Jacobian conjecture. For background see, e.g., Bull. Amer. Math. Soc. (N.S.) 7 (1982), no. 2, 287–330 [MR 83k:14028], J. Reine Angew. Math. 340 (1983), 140–212 [MR 84m:14018], and J. Pure Appl. Algebra 130 (1998), no. 3, 277–292 [MR 99g:14015].